phase does not vanish during the mixing process. In this case, too, the presence of the liquid fraction with its capacity to absorb or release large amounts of heat through the change of phase levels the T_3 curve. The differences in pressure rise between the vapor and the perfect gas are smaller than in the previous case.

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Reply by Author to G. Angelino

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I APPRECIATE Mr. Angelino's comment on the neglect of cesium vapor condensation effect which appeared to exist in some test runs in my paper, "Jet Compression for Closed-Cycle Magnetoplasmadynamic Electrical Power Generation." My paper was primarily a report of the results of an experimental investigation of the feasibility of using a stream of metallic vapor of high molecular weight to entrain and compress a gaseous stream of low molecular weight. The objective of the investigation was the experimental proof of the applicability and not a theoretical treatise of the jet compression process. Since the experimental set-up employed an expansion nozzle of fixed geometry and experiments were carried out at different stagnation temperatures and pressures of the primary driving stream, a theoretical treatment of the jet compression process has to take into consideration not only the change of state of fractions of the working medium, but also the effect of underexpansion and overexpansion of the primary stream. None of such effects can really be accounted for without extensive experimental investigations.

It is well known that a vapor stream, unless disturbed, can remain fully in the vapor state under certain degrees of cooling below its saturation point. In our experimental set-up both the helium gas and the cesium system were thoroughly purged and evacuated before experimentation so as to avoid condensation by nucleation. Experiments carried out at Massachusetts Institute of Technology Lab on condensation of CO_2 and ammonia have indicated wide discrepancies in degree of supersaturation for different vapor streams expanding through convergent-divergent nozzles.[1,2] It was also observed that the more abrupt the expansion, the higher the degree of supersaturation of the expansing stream. Presumably, the molecular weight of the medium will also affect the degree of supersaturation due to difference in diffusion velocity.

Our own experience with Cs has indicated that Cs vapor in mixture with He is very difficult to condense. This was evidenced in both the MPD electrical power generation[3] experiment and the jet compression experiment. In the former case, a straight tubular type of condenser was used. However, cesium deposits in large quantity were found passing through the cooler and condenser into the He surge tank and eventually the helium compressor. In the latter program, a multitube multibaffled-type mineral-oil-cooled condenser was used. Again a large amount of Cs was condensed in the He surge tank and the heater pipe. This strongly suggests the complexity of condensation of Cs vapor. The lack of information about condensation of Cs in supersonic nozzle flow and in mixtures of Cs with noble gases negates a realistic performance analysis without extensive experimental investigation.

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Comments on "An Improved Finite-Difference Method for Heat-Transfer Calculation"

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GROSS[1] developed an equation for the time step interval to be applied to the explicit finite-difference technique used in transient heat-transfer calculation. Briefly, his development is as follows:

$$t_i^+ = t_i + \frac{\Delta\tau_i}{C_i}\left[\sum_j K_{ji}(t_j - t_i) + S_i\right] \tag{1}$$

$$t_i^*\left(= \sum_j K_{ji}t_j + S_i\right)/\sum_j K_{ji} \tag{2}$$

where t_i^+ = estimate of temperature of node i at the "new" time $\tau + \Delta\tau_i$; t_i = temperature of node i at the "old" time τ; C_i = thermal capacity of node i; K_{ji} = thermal conductance between node i and j nodes; S_i = energy source at node i; t_i^* = temperature of node i if node i is in thermal equilibrium with neighboring nodes (j nodes); and $\Delta\tau_i$ = time step. The criterion for stability is given by Gross as:

$$t_i^+ \leq t_i^* \tag{3}$$

which he states results in the well known equation,

$$\Delta\tau_i \leq C_i/\sum_j K_{ji} \tag{4}$$

If a small error in temperature is allowed, Gross writes

$$t_i^+ \leq t_i^* + \Delta t \tag{5}$$

which he states results in the following criterion:

$$\Delta\tau_i \leq \frac{C_i}{\sum_j K_{ji}}\left[1 + \frac{\Delta t\sum_j K_{ji}}{\sum_j K_{ji}(t_j - t_i) + S_i}\right] \tag{6}$$

Note there is an obvious typographical error in Gross's paper in that the

$$\sum_j K_{ji}$$

in the second expression in the bracketed term of Eq. (6) [Eq. (10) in Gross's paper] is missing.

The objection to the development of Gross's is that the time step stability criterion, Eq. (6), can result in a time step less than that given by Eq. (4). This is possible since the

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term

$$\sum_j K_{ji}\,(t_j - t_i) + S_i$$

can be negative. Actually, Eqs. (4) and (6) do not follow from Eqs. (3) and (5) because the term

$$\sum_j K_{ji}(t_j - t_i) + S_i$$

can take on negative values that in turn reverse the directions of the inequalities. Moreover, a test is needed to examine the term

$$\sum_j K_{ji}(t_j - t_i) + S_i$$

for possible zero values. If a value of zero is encountered, either node i must be neglected or the term

$$\sum_j K_{ji}(t_j - t_i) + S_i$$

must be assigned a very small value in order that division by zero is not attempted. Also, Gross has not clarified which node in a system is the "most sensitive node." Is it the node that has the minimum value of time step per Eq. (4) or per Eq. (6)? That is, does one find the node with the minimum time step per Eq. (4) and then apply Eq. (6) to establish the compute time step, or does one apply Eq. (6) to all nodes and use the minimum value as the compute time step?

These questions can be resolved by revising and enlarging Gross's development as follows:

1) Replace Eq. (3) with

$$|t_i^+ - t_i| \leq |t_i^* - t_i| \tag{3a}$$

This results in

$$\Delta\tau_i \leq C_i/\sum_j K_{ji}$$

2) Replace Eq. (5) with

$$|t_i^+ - t_i| \leq |t_i^* - t_i| + |\Delta t| \tag{5a}$$

This results in

$$\Delta\tau_i \leq \frac{C_i}{\sum_j K_{ji}}\left[1 + \frac{\Delta t \sum_j K_{ji}}{\left|\sum_j K_{ji}(t_j - t_i) + S_i\right|}\right] \tag{6a}$$

Equation (6a) is offered as a stability criterion that will force a finite-difference solution to be bounded within an error of Δt. Also, it can be shown that one must apply Eq. (6a) to all nodes and then use the minimum value found as the compute time step. This is proven by considering node k with neighboring nodes l. If node i is taken to be the most sensitive node we write

$$t_k^+ = t_k + \left[1 + \frac{\Delta t \sum_j K_{ji}}{\left|\sum_j K_{ji}(t_j - t_i) + S_i\right|}\right] \frac{C_i/\sum_j K_{ji}}{C_k/\sum_l K_{lk}}\,(t_k^* - t_k) \tag{7}$$

$$t_k^+ = t_k\left(1 - \frac{\Delta\tau_i}{\Delta\tau_k}\right) + \frac{\Delta\tau_i}{\Delta\tau_k}\left[t_k^* + \frac{\Delta t \sum_l K_{lk}(t_l - t_k) + S_k}{\left|\sum_l K_{lk}(t_l - t_k) + S_k\right|}\right] \tag{8}$$

where $\Delta\tau_i$ and $\Delta\tau_k$ are per Eq. (6a).

Equations (7) and (8) are expressions for the "new" temperature at the arbitrary node k. These equations are derived by writing Eq. (1) for node k and substituting Eq. (6a) for $\Delta\tau_i$. Equation (2) is used to bring t_k^* into the equations and then the equations are arranged in terms of ratios of $C/\Sigma K$'s and ratios of $\Delta\tau$'s per Eq. (6a). Relative to Eq. (7), we imagine that node i has a minimum value of $C/\Sigma K$ and we find that t_k^+ is not bounded, since the term

$$\left|\sum_j K_{ji}\,(t_j - t_i) + S_i\right|$$

can take on any value. Relative to Eq. (8), we imagine that node i has a minimum value of the time step defined by Eq. (6a). Here we find that t_k^+ is bounded.

As a final comment, we wonder just how valuable Eq. (6a) is. Even though one can use larger computer time steps and retain stability, does one actually save on computer time relative to the usual attack using Eq. (4)? This question arises since Eq. (6a) must be evaluated for each node at each time step, and this certainly is more time consuming than applying Eq. (4). It is felt, for example, that an increase in computer time is certainly probable for problems that have nodal time constants ($c/\Sigma K$) very nearly equal for all of the nodes.

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Reply by Author to R. K. McMordie and L. W. Mordock

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MCMORDIE and Mordock offer a similar but more rigorous development, since their starting point, Eq. (3a), is an exact representation of the statement preceding Eq. (4) in Ref. 1. The final results, however, are the same. Equation (6a) is equivalent to Eq. (10) of Ref. 1, for Δt may be either positive or negative, and will have the same sign as the term

$$\sum_j K_{ji}(t_j - t_i) + S_i$$

Thus, computing time steps cannot be less than that given by Eq. (4) [Eq. (7) of Ref. 1]. The only advantage that Eq. (10) of Ref. 1 has over Eq. (6a) is that, by keeping track of signs, different criteria could be used for rising and falling temperatures.

McMordie and Mordock are unduly concerned over possible zero values for the term

$$\sum_j K_i(t_{ij} - t_i) + S_i$$

It is common practice to place an upper limit on the time step in order to evaluate subroutines and boundary conditions frequently enough, as well as to assure that calculations are spaced closely enough to adequately describe the results.

Equations (7) and (8) fail to prove that the same computing time step must be applied to all nodes. I have solved problems where the same computing time step was not used for all nodes, and have found, as expected, that this can save machine time.

Concern over more machine time to compute Eqs. (6a) than Eq. (4) is not warranted in most problems. The extra terms in Eq. (6a) which are not in Eq. (4) have already been evaluated for calculating temperature, so relatively few extra computations are required. Thus, computation time for problems is approximately inversely proportional to the computing time step.

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